GCE AS/A level

0985/01

S15-0985-01

MATHEMATICS – S3
Statistics

A.M. MONDAY, 22 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A bag contains 6 balls numbered 1, 2, 2, 4, 6, 6 respectively. Three of these balls are selected at random, without replacement, and X_1, X_2, X_3 denote the numbers on these balls written in ascending order. The sample range R is defined by $R = X_3 - X_1$ and the sample median M is defined by $M = X_2$.

Determine the sampling distributions of R and M. [8]

2. Emlyn solves the crossword in the Daily Bugle every day. He records the time taken to do this on 12 randomly chosen days with the following results (in minutes).

16·3 17·4 14·3 15·6 16·4 13·9 16·9 17·4 17·9 15·3 16·6 14·9

You may assume that these times are normally distributed with mean μ and variance σ^2.

(a) Calculate unbiased estimates of μ and σ^2. [5]

(b) Determine a 99% confidence interval for μ. [5]

3. A farmer grows two varieties of apples, A and B. He wishes to investigate whether or not there is a difference in the mean weights of apples of these two varieties. He therefore selects and weighs a random sample of 100 apples of each variety with the following results.

	Variety A	Variety B
Sample size	100	100
Sample mean (grams)	160·53	161·17
Unbiased variance estimate (grams^2)	7·62	6·91

(a) State suitable hypotheses for the investigation. [1]

(b) Calculate the approximate *p*-value of the above results and state your conclusion in context. [8]

4. A large forest is populated by squirrels, of which some are red and some are grey.

(a) A zoologist wishes to determine the proportion, *p*, of red squirrels in the forest. She therefore traps a random sample of 90 squirrels and finds that 54 of them are red. Determine an approximate 90% confidence interval for *p*. [6]

(b) A colleague decides that this interval is too wide. He traps a random sample of squirrels and calculates the following approximate 95% confidence interval for *p*.

[0·5445, 0·6485]

Determine

(i) the sample size,

(ii) the number of red squirrels in the sample. [6]

5. The speed of sound in air, y ms^{-1}, and the air temperature, x °C, may be assumed to be related by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants α and β, the following measurements were made.

x	10	15	20	25	30
y	337·1	340·7	343·0	346·1	349·7

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·25.

(i) Determine a 99% confidence interval for α, giving your answer correct to one decimal place. [5]

(ii) Test, at the 5% significance level, the null hypothesis $H_0 : \beta = 0{\cdot}65$ against a two-sided alternative. [7]

6. The discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	θ	2θ	3θ	$1 - 6\theta$

where θ is an unknown constant, $0 < \theta < \frac{1}{6}$. In order to estimate θ, a random sample of n observations on X is obtained and $\bar{X}$ denotes the sample mean.

(a) Given that $U = a\bar{X} + b$ is an unbiased estimator for θ, determine

(i) the constants a and b,

(ii) the variance of U. [9]

(b) Let Y denote the number of observations in the sample equal to 4. Given that $V = cY + d$ is an unbiased estimator for θ, determine

(i) the constants c and d,

(ii) the variance of V. [5]

(c) Show that

$$\frac{\mathrm{Var}(U)}{\mathrm{Var}(V)} = \frac{6 - 30\theta}{5 - 30\theta}.$$

Hence, giving a reason, state which of U and V is the better estimator for θ. [2]

END OF PAPER